APC Project: SimpleQuadTree

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Introduction

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Original target:

Aim 1

Construct **highly adaptive Cartesian non-uniform meshes** on which we can perform **numerical quadrature** and eventually implement **numerical solvers for PDEs**, mainly based on the Finite Volume method.

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Aim 2

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These two objectives may seems quite far one from the other but actually they can be linked by... **Quadtrees**.

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Basically just an unbalanced tree structure allowing **four children**. Generalized in 3D by the octrees.

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Basically just an unbalanced tree structure allowing **four children**. Generalized in 3D by the octrees. It has a clear geometrical counterpart in terms of AMR (Adaptive Mesh Refinement), where they are built by **recursively splitting** larger cells.

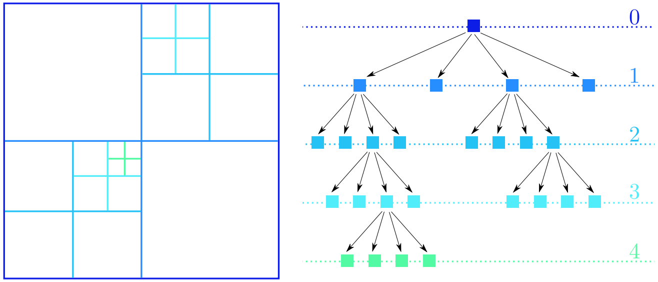

Image from: T. Bellotti, M. Theillard - A coupled level-set and reference map method for interface representation with applications to two-phase flows simulation - Volume 392, 2019, J. Comput. Phys.

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- **Level** (of a cell): number of times the largest cell has been split to generate the current cell.
- **Maximum level** ($\max_{\text{level}}$): maximum number of allowed splits.
- **Minimum level** ($\min_{\text{level}}$): minimum number of necessary splits.

Models and tools to understand what follows

Level-set theory

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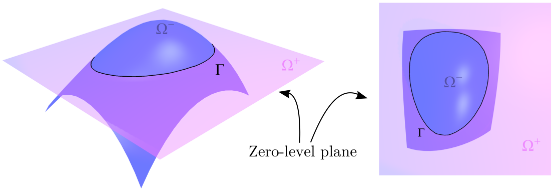

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$$\begin{aligned}\Gamma &= \{\mathbf{x} \in \Omega \ : \ \phi(\mathbf{x}) = 0\}\,,\\ \Omega^- &= \{\mathbf{x} \in \Omega \ : \ \phi(\mathbf{x}) < 0\}\,,\\ \Omega^+ &= \{\mathbf{x} \in \Omega \ : \ \phi(\mathbf{x}) > 0\}\,.\end{aligned}$$

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It is probably the easiest way of **representing an interface** in a computationally efficient manner.

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In 2D, a circle centered in (x_0, y_0) with radius R can be represented by:

$$\phi(x, y) = \sqrt{(x - x_0)^2 + (y - y_0)^2} - R.$$

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Notice that, given Γ, the level-set is not unique, but this is not our problem now. It is unique if we assume (and we will do so) that $\phi(x, y)$ is nothing but the **signed distance** of (x, y) from Γ.

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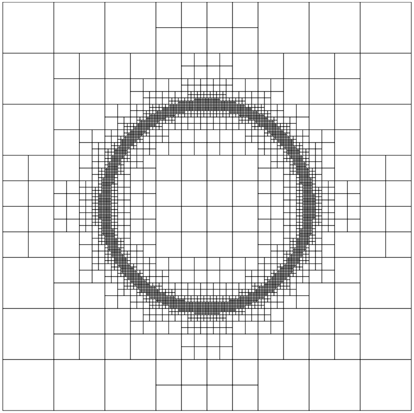
$$\text{split } \mathcal{C} \text{ if } \quad : \quad |\phi(\mathbf{x}_\mathcal{C})| \leq \mathrm{Lip}(\phi)\cdot \mathrm{diag}(\mathcal{C}) \qquad \text{and} \qquad \mathrm{level}(\mathcal{C}) \leq \max_{\mathrm{level}},$$

This naively means that we split a cell wheather its diagonal length exceeds the distance from the interface Γ represented by the level-set ϕ.

And the result can be. . . Very nice!



Gaussian quadrature

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Considering a stardard quadrilateral domain $\Omega = [-1, 1]^2$ and given a possibly smooth function $f : \Omega \to \mathbb{R}$, we want to compute:

$$\int_\Omega f(x, y) dx dy.$$

This can be done by quadrature formulae, based on simple evaluation of the function f at certain points of the domain. For our purpose, we use two choices:

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2. **3rd order Gaussian formula**, with nine function evaluations (plus extra computations to adapt to a generic domain).

$$\begin{aligned}\int_\Omega f(x,y)dxdy \simeq \frac{5}{81}&\left[5f\left(-\sqrt{\frac{3}{5}},-\sqrt{\frac{3}{5}}\right)+8f\left(0,-\sqrt{\frac{3}{5}}\right)+5f\left(\sqrt{\frac{3}{5}},-\sqrt{\frac{3}{5}}\right)\right]\\ &+\frac{8}{81}\left[5f\left(-\sqrt{\frac{3}{5}},0\right)+8f(0,0)+5f\left(0,\sqrt{\frac{3}{5}}\right)\right]\\ &+\frac{5}{81}\left[5f\left(-\sqrt{\frac{3}{5}},\sqrt{\frac{3}{5}}\right)+8f\left(0,\sqrt{\frac{3}{5}}\right)+5f\left(\sqrt{\frac{3}{5}},\sqrt{\frac{3}{5}}\right)\right].\end{aligned}$$

Rgb colors and their “distance”

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It is probably the simplest way of representing colors, as a combination of three components, **red, green and blue**. Thus, a color C is nothing else than a triplet:

$$C = [C_{\text{red}}, C_{\text{green}}, C_{\text{blue}}],$$

where $C_{\text{red}}, C_{\text{green}}, C_{\text{blue}} \in \{0, \ldots, 255\}$. In this way, we can represent 16,777,216 different tones.

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$$d(C^1, C^2) = \sqrt{2\left(C^1_{\text{red}} - C^2_{\text{red}}\right)^2 + 4\left(C^1_{\text{green}} - C^2_{\text{green}}\right)^2 + 3\left(C^1_{\text{blue}} - C^2_{\text{blue}}\right)^2}.$$

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The mean color between $\{C^1, \dots, C^N\}$ is defined by:

$$\overline{C}\left(\left\{C^1, \dots, C^N\right\}\right) = \left[\frac{1}{N}\sum_{n=1}^{N} C^n_{\text{red}}, \frac{1}{N}\sum_{n=1}^{N} C^n_{\text{green}}, \frac{1}{N}\sum_{n=1}^{N} C^n_{\text{blue}}\right],$$

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and a sort of standard deviation as:

$$\sigma\left(\left\{C^1, \ldots, C^N\right\}\right) = \frac{1}{N}\sqrt{\sum_{n=1}^{N} d(C_n, \overline{C})^2} \in [0, 765].$$

How to compress an image?

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Let $\mathcal{P}$ be a preleave whose children are $\mathcal{L}(\mathcal{P})$. We define a tolerance $0 < \epsilon \ll 1$. Then:

$$\text{merge} \quad \mathcal{P} \quad \text{if} \ : \ \sigma\left(\mathcal{L}(\mathcal{P})\right) \leq 765 \cdot \epsilon.$$

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In the case of merging, we consider:

$$C_{\mathcal{P}} = \overline{C}\left(\mathcal{L}(\mathcal{P})\right).$$

Implementation

The AbstractCell class

The AbstractCell class

	AbstractCell<T>
#	const Point<T> base_point
#	const T dx
#	const T dx
#	const unsigned char level
#	std::shared_ptr<AbstractCell> l_l
#	std::shared_ptr<AbstractCell> l_r
#	std::shared_ptr<AbstractCell> u_l
#	std::shared_ptr<AbstractCell> u_r
+	AbstractCell(Point<T>, T, T, unsigned char)
+	virtual ~AbstractCell()
+	Point<T> getBasePoint() const
+	unsigned char getLevel() const
+	bool isLeaf() const
+	Point<T> getCenter() const
+	T getDx() const
+	T getDy() const
+	virtual void splitCell() = 0
+	virtual void mergeCell()
+	void refineCell(const RefinementCriterion<T> & , const unsigned char)
+	void simplifyCell(const RefinementCriterion<T> &, const unsigned char)
+	void updateCell(const RefinementCriterion<T> &, const unsigned char, const unsigned char)
+	std::vector<std::shared_ptr<AbstractCell<T>>> getChildren() const
+	std::vector<Point<T>> getVertices() const
+	void getLeaves(std::vector<std::shared_ptr<AbstractCell<T>>> &)
+	void getPreLeaves(std::vector<std::shared_ptr<AbstractCell<T>>> &) const
+	std::string tikzDot() const
	std::string tikzSquare(const RGBColor color, const bool) const

Basic features of a cell in a quadtree.

The Cell class inheriting from AbstractCell

The Cell class inheriting from AbstractCell

	Cell<T> : public AbstractCell<T>
+	Cell(Point<T>, T, T, unsigned char)
+	virtual ~Cell()
+	T getDiagonal() const
+	T cellSurface() const
+	virtual void splitCell() override
+	T zeroOrderIntegration(const std::function<T(Point<T>)> &) const
+	T thirdOrderGaussianIntegration(const std::function<T(Point<T>)> &) const

Used to build the quadtree on which we construct a mesh for performing **quadratures**. These functions are not necessary to do image compression.

The Pixel class inheriting from AbstractCell

The Pixel class inheriting from AbstractCell

The “generalized” pixel is basically an AbstractCell...

	Pixel<T> : public AbstractCell<T>
#	`RGBColor field`
+	`Pixel(Point<T>, T, T, unsigned char)`
+	`Pixel(Point<T>, T, T, unsigned char, const RGBColor &)`
+	`virtual ~Pixel()`
+	`void setField(const RGBColor &)`
+	`RGBColor getField() const`
+	`virtual void splitCell() override`
+	`RGBColor meanField()`
+	`double stdDevField()`
+	`virtual void mergeCell() override`

plus a **color** and functions to compute color means and standard deviations. We have to be careful to **cast pointers** when we need to extract information from the Pixel.

The QuadTree class

The QuadTree class

This class is mostly a **wrapper** of the Cell class, but it is what the user interacts with.

	QuadTree <T>
#	const T x_size
#	const T y_size
#	const unsigned char min_level
#	const unsigned char max_level
#	std::shared_ptr<Cell<T>> parent_cell
#	std::vector<std::shared_ptr<Cell<T>>> getLeaves() const
+	QuadTree(Point<T>, T, T, unsigned char, unsigned char)
+	virtual ~QuadTree()
+	T simpleIntegration(std::function<T(Point<T>)> &) const
+	unsigned getMinLevel() const
+	unsigned getMaxLevel() const
+	size_t numberOfLeaves() const
+	void buildUniform()
+	void buildUniform(unsigned)
+	void clear()
+	void updateWithLevelSet(const LipschitzFunction<T> &)
+	void updateQuadTree(const RefinementCriterion<T> &)
+	void updateQuadTree(const RefinementCriterion<T> &, const unsigned char, const unsigned char)
+	std::vector<Point<T>> getCenters()
+	void exportCentersTikz(const std::string &) const
+	void exportMeshTikz(const std::string &, bool) const
+	T simpleIntegration(const std::function<T(Point<T>)> &) const
+	T thirdOrderGaussianIntegration(const std::function<T(Point<T>)> &) const
+	T simpleIntegration(const std::function<T(std::shared_ptr<Cell<T>>)> &) const

The Image class

The Image class

This class is mostly a **wrapper** of the Pixel class. We distinguished it from the QuadTree class (it does not inherit from it) because there are many features that they do not share.

	Image<T>
#	T x_size
#	T y_size
#	unsigned char min_level
#	unsigned char max_level
#	std::shared_ptr<Pixel<T>> parent_cell
+	Image()
+	Image(T, T, unsigned char, unsigned char)
+	~Image()
+	unsigned int getMinLevel() const
+	unsigned int getMaxLevel() const
+	size_t numberOfPixels() const
+	void clear()
+	void simplifyImage(double)
+	void buildUniform(unsigned char)
+	void createFromFile(const std::string &)
+	void saveImage(const std::string &) const

The way we update the mesh: the RefinementCriterion class

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This is a very simple **abstract class** with an operator telling us if we have to split an AbstractCell or not.

	RefinementCriterion <T>
+	RefinementCriterion()
+	virtual ~RefinementCriterion()
+	virtual bool operator()(std::shared_ptr<AbstractCell<T>>) const = 0

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	RefinementCriterion <T>
+	RefinementCriterion()
+	virtual ~RefinementCriterion()
+	virtual bool operator()(std::shared_ptr<AbstractCell<T>>) const = 0

Many important criteria inherit from it:

	RefineAlwaysCriterion<T> : public RefinementCriterion<T>
+	RefineAlwaysCriterion()
+	virtual ~RefineAlwaysCriterion()
+	virtual bool operator()(std::shared_ptr<AbstractCell<T>>) const override

	LevelSetCriterion<T> : public RefinementCriterion<T>
-	const LipschitzFunction<T> & level_set
+	LevelSetCriterion(const LipschitzFunction<T> &)
+	virtual ~LevelSetCriterion()
+	virtual bool operator()(std::shared_ptr<AbstractCell<T>>) const override

	CriterionVariance<T> : public RefinementCriterion<T>
-	double thr
+	CriterionVariance()
+	virtual ~CriterionVariance()
+	virtual bool operator()(std::shared_ptr<AbstractCell<T>>) const override

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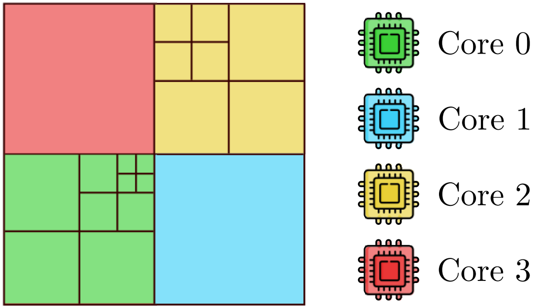
The key idea is to have a number of core which is a power of 4 and have a minimum level large enough, so that we can avoid communications between cores and each of them has a local tree (no shared memory).



And each subtree (core) integrates independently. At the very end, the sub-integrals are summed with a **reduce** procedure.

Tests and results

We are able to construct very general meshes

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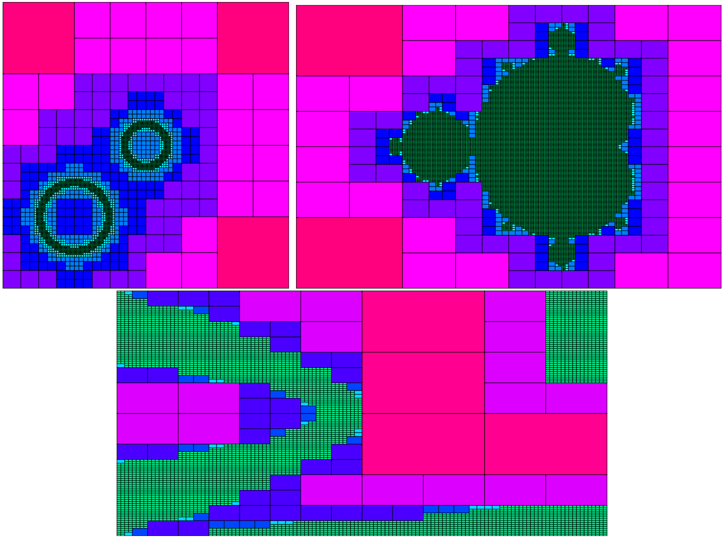

We are able to perform parallel quadratures

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Computations have been tested on **4 cores** with the following specifications:

- Product: Intel(R) Xeon(R) CPU E5-2673 v4 @ 2.30GHz

Taking:

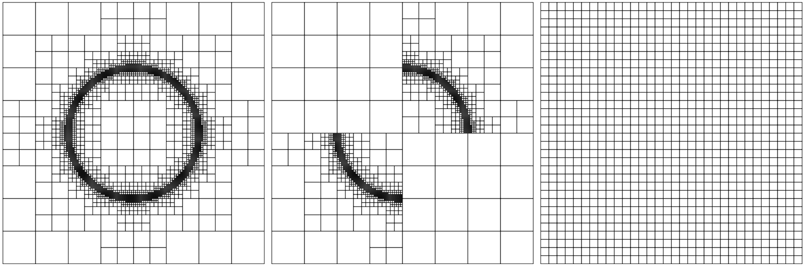
$$\min_{\text{level}} = 3 \qquad \max_{\text{level}} = 11,$$

so, for the uniform mesh, we deal with

$$2^{11} \times 2^{11} \quad \text{cells} = 4'194'304 \quad \text{cells}.$$

Each time, we perform two tests and take the average time in order to avoid spurious effects.

$$\Omega = [-2,2]^2 \quad \phi(x,y) = \sqrt{x^2+y^2} - 1 \quad f(x,y) = \mathbb{I}_{\{\phi(x,y)\le 0\}} \quad \int_\Omega f(x,y)dxdy = \pi.$$

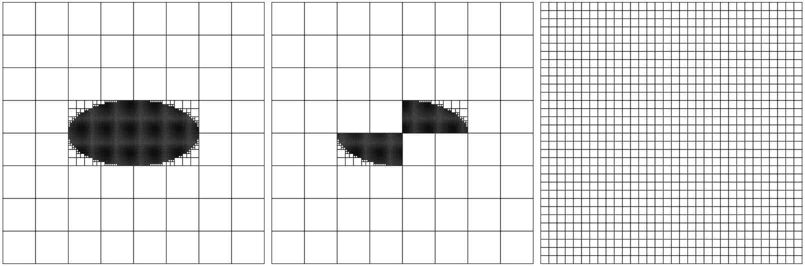


Naive	Mesh 1		Mesh 2		Mesh 3	
# of cores	Time [s]	Speedup	Time[s]	Speedup	Time [s]	Speedup
1	6.585	-	6.639	-	7.279	-
4	1.721	**3.826**	1.774	**3.742**	1.912	**3.807**

3rd Gaussian	Mesh 1		Mesh 2		Mesh 3	
# of cores	Time [s]	Speedup	Time[s]	Speedup	Time [s]	Speedup
1	7.138	-	6.859	-	73.234	-
4	1.856	**3.846**	1.917	**3.578**	18.980	**3.858**

$$\Omega = [-2,2] \quad f(x,y) = \frac{1}{2\pi\sigma_x\sigma_y} e^{-\frac{1}{2}\left(\frac{x^2}{\sigma_x^2}+\frac{y^2}{\sigma_y^2}\right)} \quad \sigma_x = 0.1 \quad \sigma_y = 0.05 \quad \int_\Omega f(x,y)dxdy \simeq 1.$$

We refine in the ellipse within 10 standard deviations.

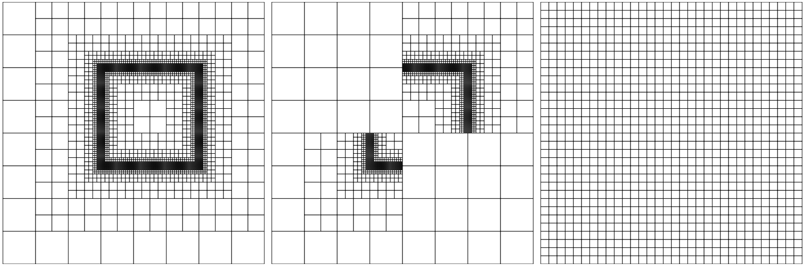


Naive	Mesh 1		Mesh 2		Mesh 3	
# of cores	Time [s]	Speedup	Time[s]	Speedup	Time [s]	Speedup
1	7.376	-	7.061	-	7.397	-
4	1.922	**3.838**	1.975	**3.575**	2.007	**3.686**

3rd Gaussian	Mesh 1		Mesh 2		Mesh 3	
# of cores	Time [s]	Speedup	Time[s]	Speedup	Time [s]	Speedup
1	13.873	-	10.410	-	74.364	-
4	3.633	**3.819**	3.679	**2.830**	19.418	**3.830**

$$\Omega = [-2,2]^2 \quad \phi(x,y) = \max\{|x-0.25|-0.75, |y-0.25|-0.75\}$$

$$f(x,y) = (x^2+y^2)\left[\cos(\pi x)+\sin(\pi y)\right]\mathbb{I}_{\{\phi(x,y)\leq 0\}} \quad \int_\Omega f(x,y)\,dx\,dy = \frac{3(-16-12\pi+7\pi^2)}{8\pi^3} \simeq 0.18610$$

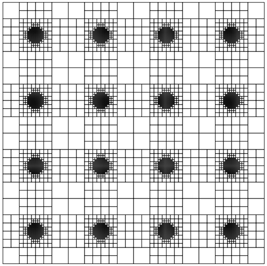
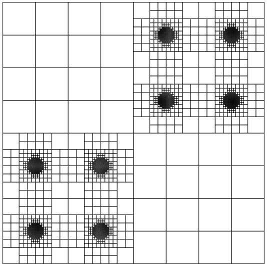
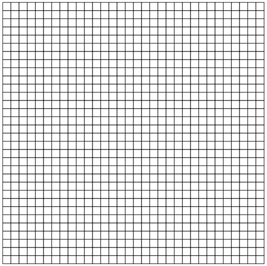


Naive	Mesh 1		Mesh 2		Mesh 3	
# of cores	Time [s]	Speedup	Time[s]	Speedup	Time [s]	Speedup
1	6.426	-	6.634	-	7.532	-
4	1.686	**3.811**	1.757	**3.776**	1.948	**3.867**

3rd Gaussian	Mesh 1		Mesh 2		Mesh 3	
# of cores	Time [s]	Speedup	Time[s]	Speedup	Time [s]	Speedup
1	7.057	-	6.914	-	75,551	-
4	1.902	**3.710**	1.987	**3.480**	20.245	**3.732**

$$\Omega = [0,8]^2 \quad \phi(x,y) = \min_{i,j=0,\dots,3}\left(\sqrt{(x-(2i+1))^2+(y-(2j+1))^2} - 0.15\right) \quad f(x,y) = \mathbb{I}_{\{\phi(x,y)\leq 0\}}$$

$$\int_\Omega \psi(x,y)dxdy = \frac{9\pi}{25}$$

Naive	Mesh 1		Mesh 2		Mesh 3	
# of cores	Time [s]	Speedup	Time[s]	Speedup	Time [s]	Speedup
1	8.145	-	7.445	-	12.063	-
4	2.146	**3.795**	2.186	**3.406**	3.167	**3.809**

3rd Gaussian	Mesh 1		Mesh 2		Mesh 3	
# of cores	Time [s]	Speedup	Time[s]	Speedup	Time [s]	Speedup
1	9.165	-	7.956	-	116.295	-
4	2.375	**3.859**	2.449	**3.249**	30.707	**3.787**

We observe that:
The inhomogeneity of the mesh does not play a very huge role since the time needed to construct and refine the mesh dominates over the time needed to integrate on it. Nevertheless, where the inhomogeneity is really strong, we observe the most important differences.

Three different ϵ: $\epsilon = 0.012, 0.024, 0.048$.

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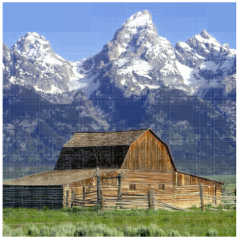
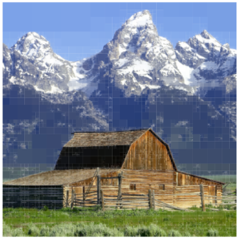
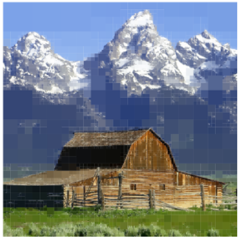
Original figures size: 262144 px.

131509 px - comp. ratio = 1.99   84808 px - comp. ratio = 3.09   38152 px - comp. ratio = 6.87

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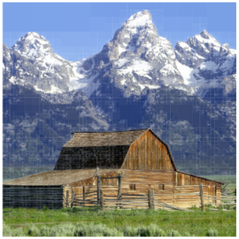
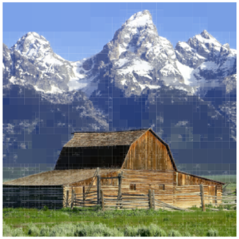
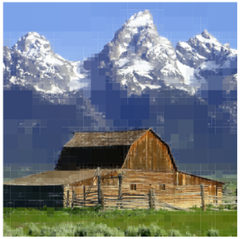

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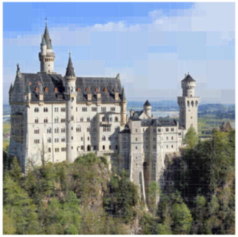
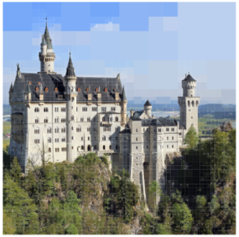
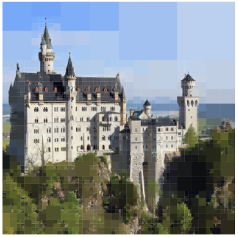
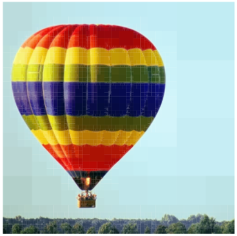
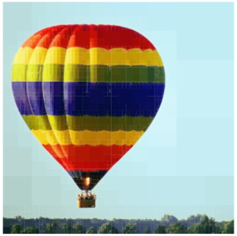
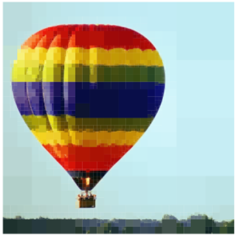
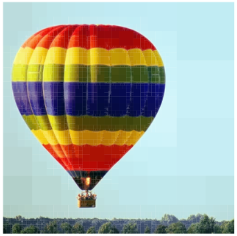
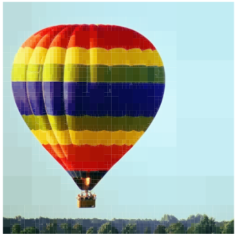
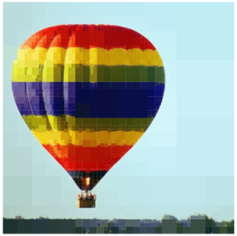




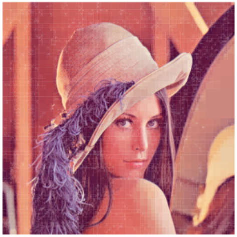
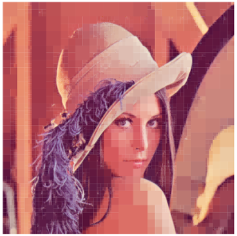
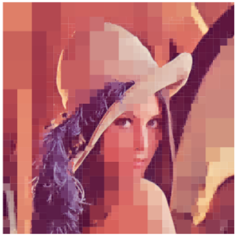
115438 px - comp. ratio = 2.27 69259 px - comp. ratio = 3.78 27394 px - comp. ratio = 9.57

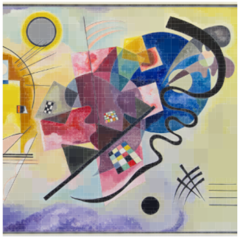
32188 px - comp. ratio = 8.14   16405 px - comp. ratio = 15.98   6571 px - comp. ratio = 39.89

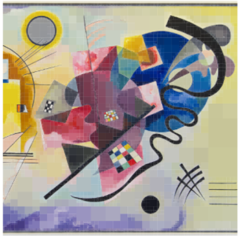




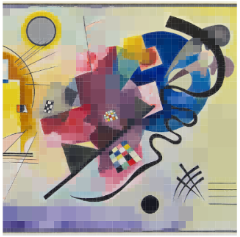
32188 px - comp. ratio = 8.14 16405 px - comp. ratio = 15.98 6571 px - comp. ratio = 39.89

75172 px - comp. ratio = 3.49 29890 px - comp. ratio = 8.77 8902 px - comp. ratio = 29.45


171517 px - comp. ratio = 1.53


85258 px - comp. ratio = 3.07


24466 px - comp. ratio = 10.71

Conclusions and perspectives

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- We are capable of **integrating in a parallel fashion** on the quadtree using **different quadrature rules**.
- We can perform **image compression** in a naive way based on the **color variance**, which significantly reduces the size of the images.

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- **Use external libraries** to import and export more "user-friendly" image formats (see .png)

Thank you!